Theory

Objective

1. Understand basic digital arithmetic using adders and subtractors.

In digital arithmetic, basic operations like addition, subtraction, multiplication, and division are all based on **binary**. These basic arithmetic operations are accomplished by using digital circuits called **adders** and **subtractors**.

1. Half Adder and Full Adder

In digital arithmetic, an adder is used to perform addition between 2 2-bit binary numbers. The half adder performs addition between bits `A` and `B`, producing outputs **`S` (sum) and `C` (carry). **

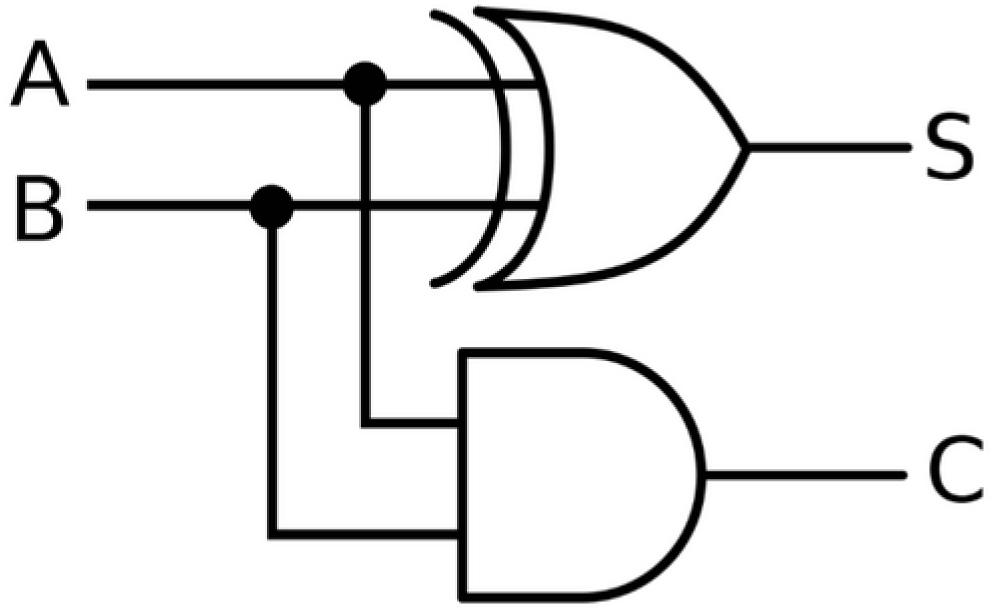

Figure 1-1: Half-Adder

Half-Adder Truth Table:

A	B	S	C
0	0	0	0

A	B	S	C
0	1	1	0
1	0	1	0
1	1	0	1

The simple adder adds the two bits together and represents them in **sum (2^0)** and **carry (2^1)**. Using this truth table, the half adder can be designed using Karnaugh maps, where the output `S` follows the logic of the XOR gate, and `C` follows the logic of the AND gate. In the half adder, we only consider inputs `A` and `B`.

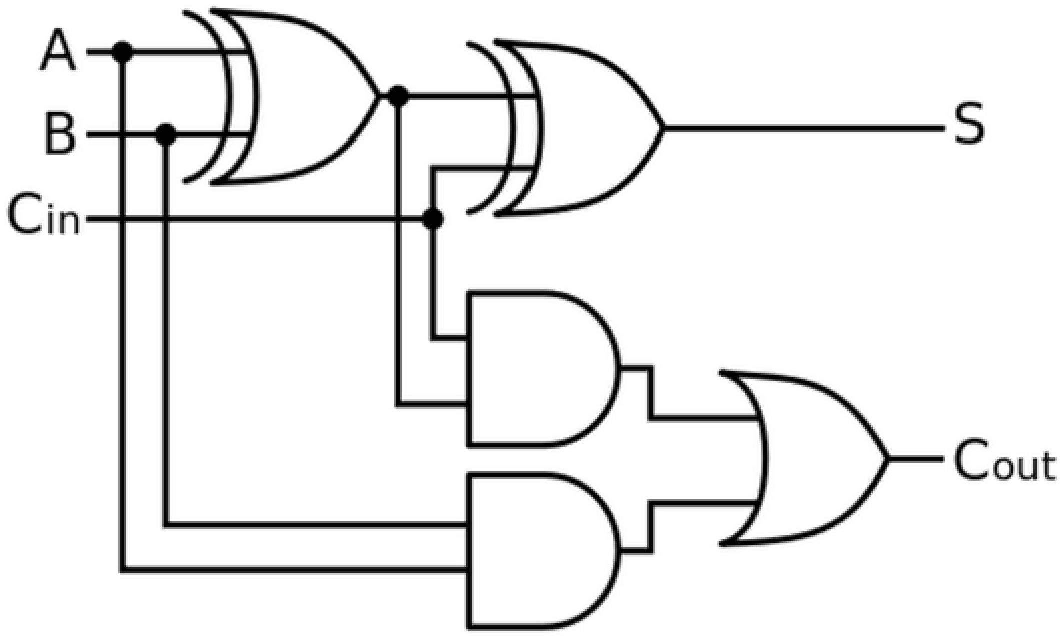

Figure 1-2: Full Adder

Truth Table for Full Adder:

A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

The full adder is a more advanced version of the half-adder, having added **carry-in** and **carry-out** parts to accomodate **higher level additions**. By using full adders, a **Ripple Carry Adder** can be created by connecting `Cout` from the less significant bit to `Cin` of the more significant bit.

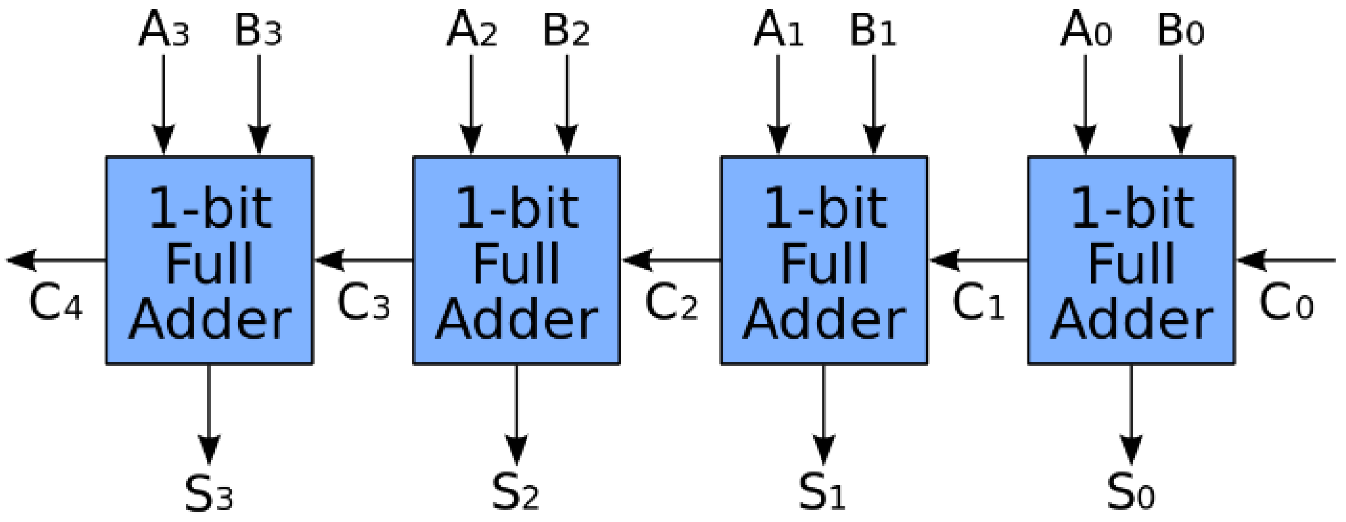

Figure 1-3: Ripple Carry Adder. We can analogize this as a chain of adders, each working on a different **power of two** (such as 2^3 <- 2^2 <- 2^1 <- 2^0).

2. Half and Full Subtractor

Opposite to adders, the subtractor is used to perform binary subtraction between 2 bits. A half subtractor performs subtraction between `A` and `B`, producing outputs `D` (difference) and `Bo` (borrow).

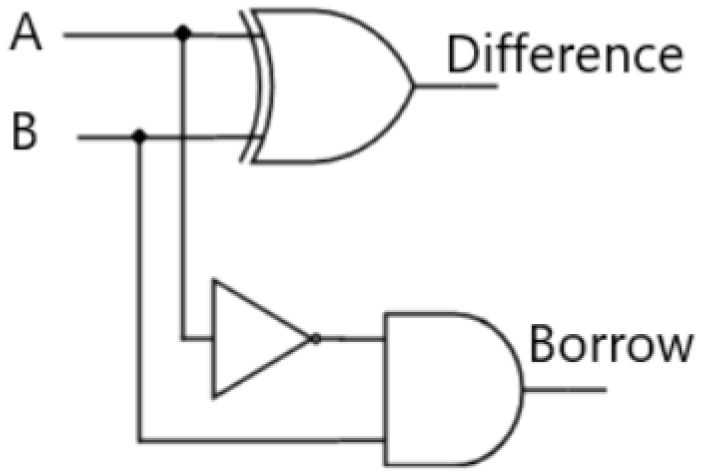

Figure 2-1: Half Subtractor

Truth Table for Half Subtractor:

A	B	D	Bo

0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Here, through boolean simplification / K-Map, `D` **follows the same logic as the XOR** gate, and `Bo` **follows the logic** `A' . B`.

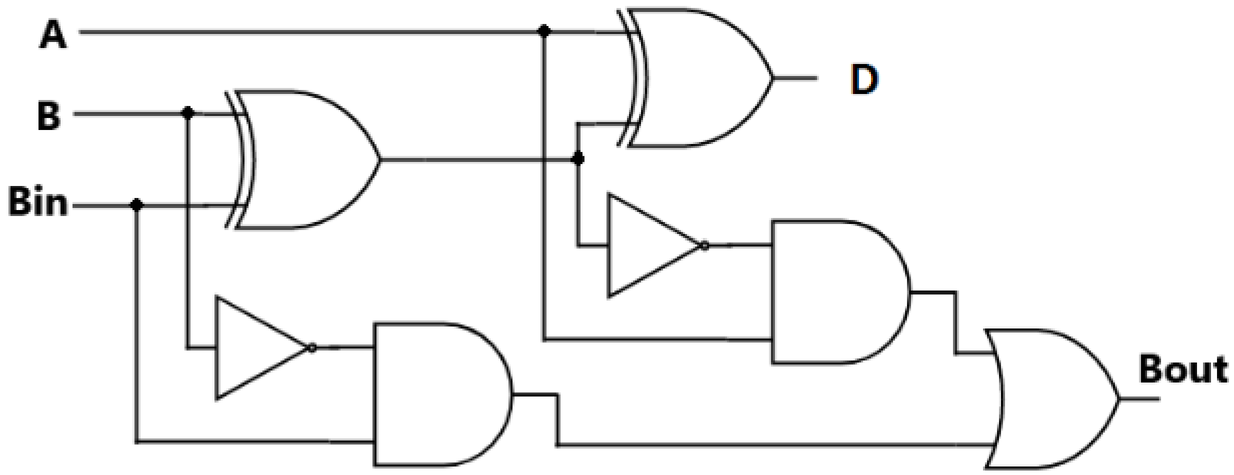

Figure 2-2: Full Subtractor

Truth Table for Full Subtractor:

A	B	Bin	D	Bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Like the full adder, the full subtractor has an additional input, `Bin` (borrow in), which comes from the `Bout` of another full subtractor. By using full subtractors, a **Ripple Borrow Subtractor** can be created by connecting `Bout` from the less significant bit to `Bin` of the more significant bit.

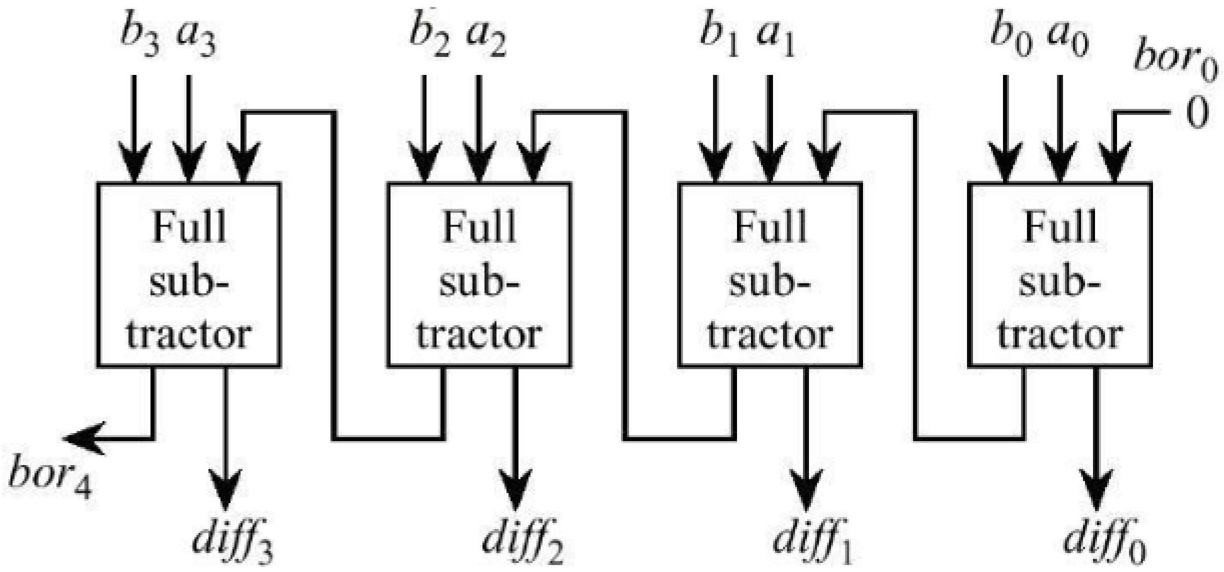

Figure 2-3: Ripple Borrow Subtractor

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