

# Theory

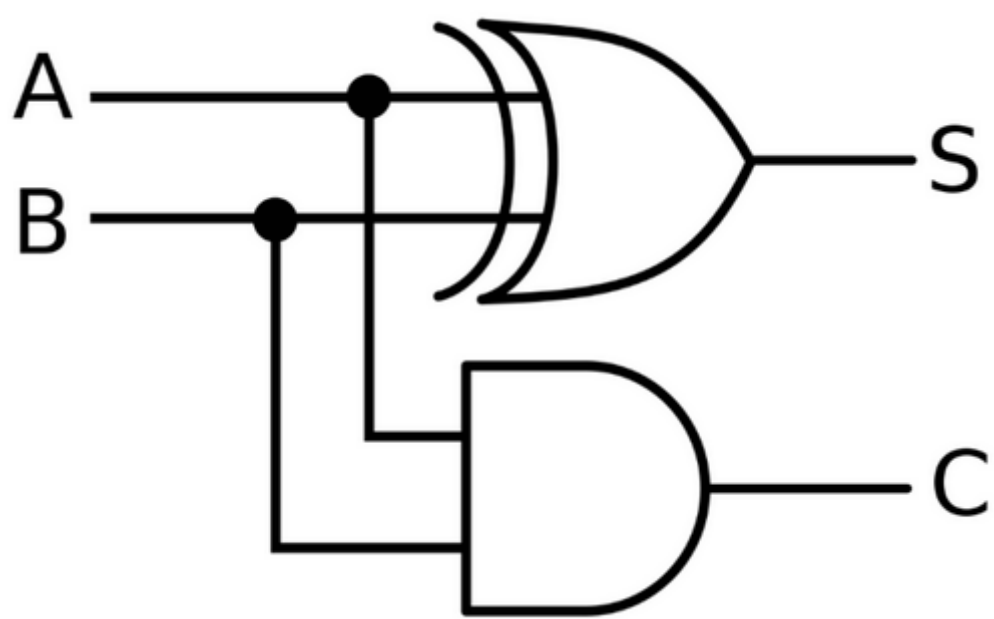
## Objective

- 1. Understand basic digital arithmetic using adders and subtractors.

In digital arithmetic, basic operations like addition, subtraction, multiplication, and division are all based on **binary**. These basic arithmetic operations are accomplished by using digital circuits called **adders** and **subtractors**.

### 1. Half Adder and Full Adder

In digital arithmetic, an adder is used to perform addition between 2 2-bit binary numbers. The half adder performs addition between bits **A** and **B**, producing outputs **S** (sum) and **C** (carry). \*\*



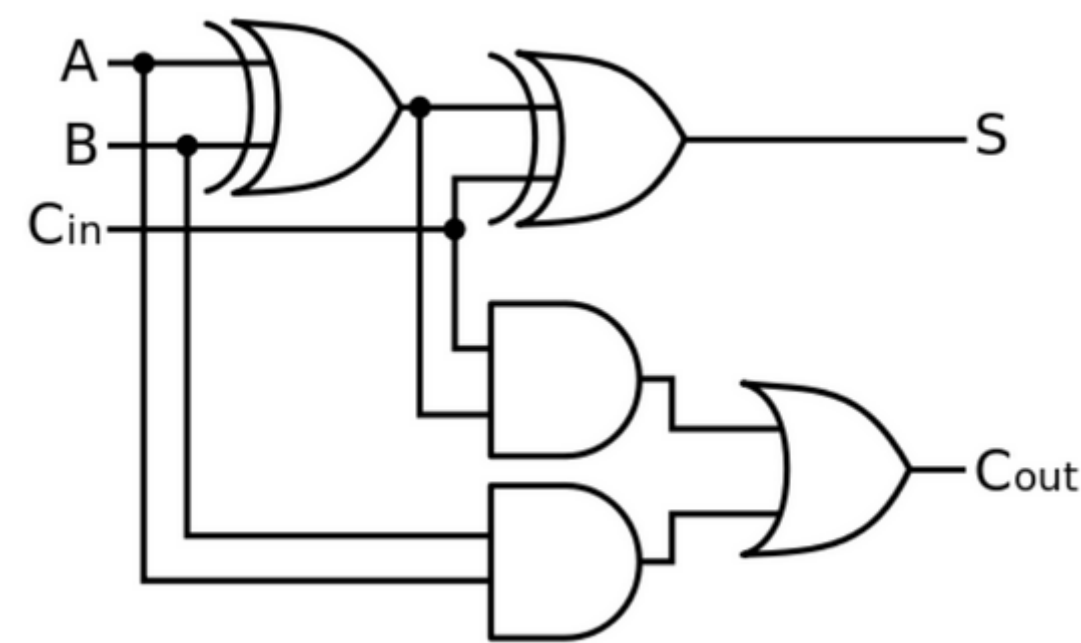
**Figure 1-1:** Half-Adder

Half-Adder Truth Table:

A	B	S	C
0	0	0	0

A	B	S	C
0	1	1	0
1	0	1	0
1	1	0	1

The simple adder adds the two bits together and represents them in **sum ( $2^0$ )** and **carry ( $2^1$ )**. Using this truth table, the half adder can be designed using Karnaugh maps, where the output **S** follows the logic of the XOR gate, and **C** follows the logic of the AND gate. In the half adder, we only consider inputs **A** and **B**.

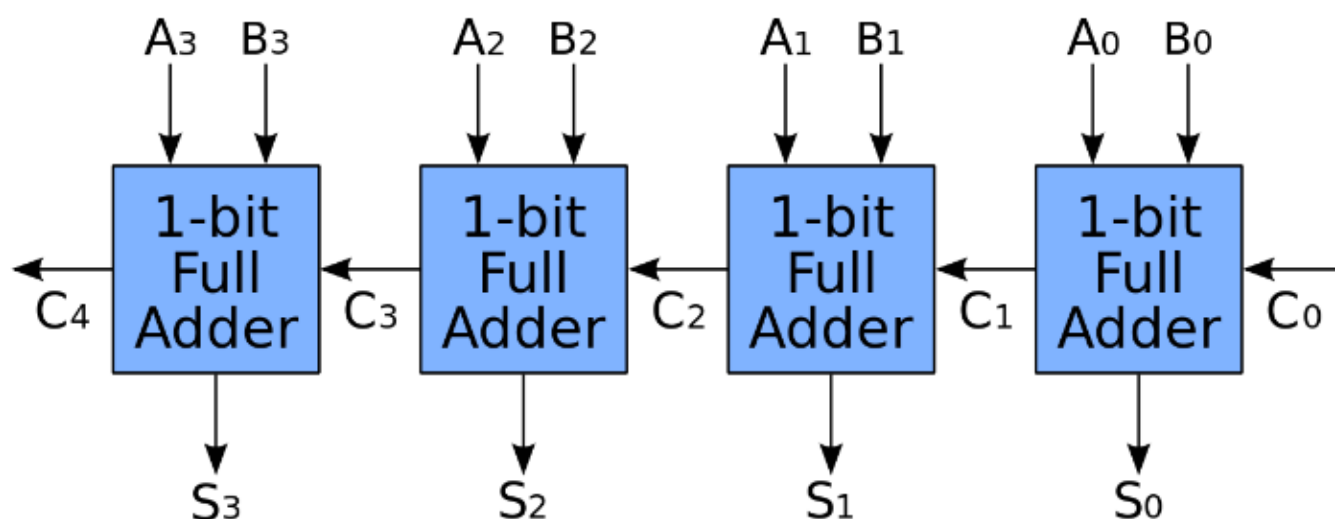


**Figure 1-2:** Full Adder

### Truth Table for Full Adder:

A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

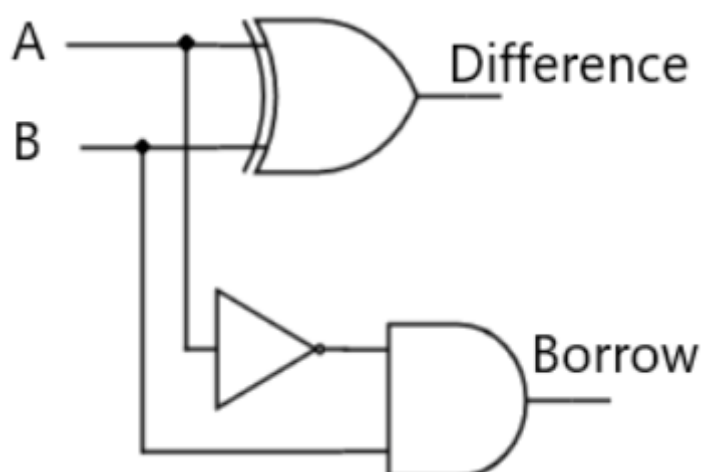
The full adder is a more advanced version of the half-adder, having added **carry-in** and **carry-out** parts to accomodate **higher level additions**. By using full adders, a **Ripple Carry Adder** can be created by connecting **Cout** from the less significant bit to **Cin** of the more significant bit.



**Figure 1-3:** Ripple Carry Adder. We can analogize this as a chain of adders, each working on a different **power of two** (such as  $2^3 <- 2^2 <- 2^1 <- 2^0$ ).

## 2. Half and Full Subtractor

Opposite to adders, the subtractor is used to perform binary subtraction between 2 bits. A half subtractor performs subtraction between **A** and **B**, producing outputs **D** (difference) and **Bo** (borrow).



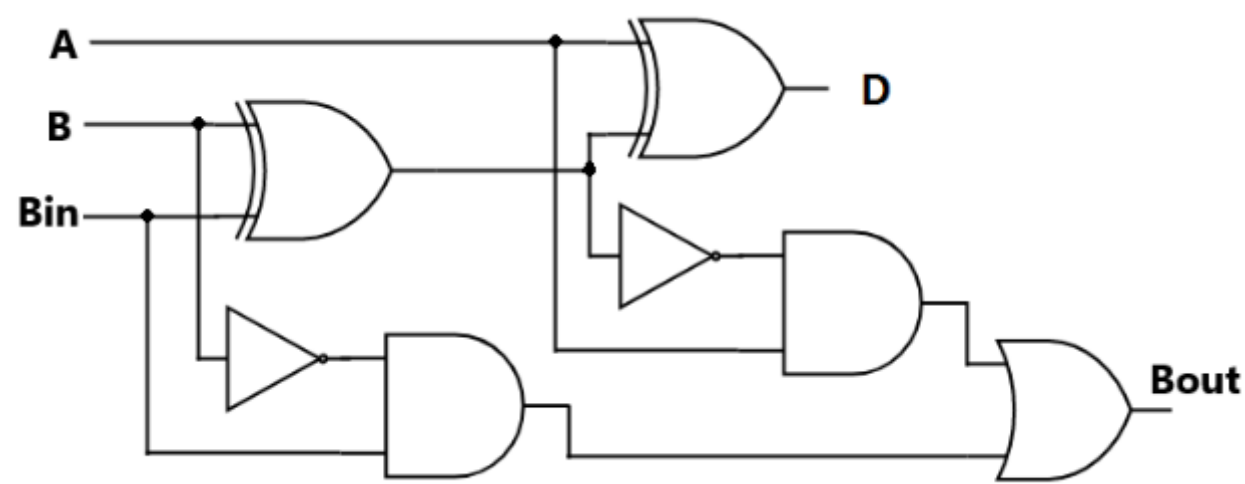
**Figure 2-1:** Half Subtractor

Truth Table for Half Subtractor:

A	B	D	Bo
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0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Here, through boolean simplification / K-Map, **D** follows the same logic as the **XOR** gate, and **Bo** follows the logic **A' . B**.

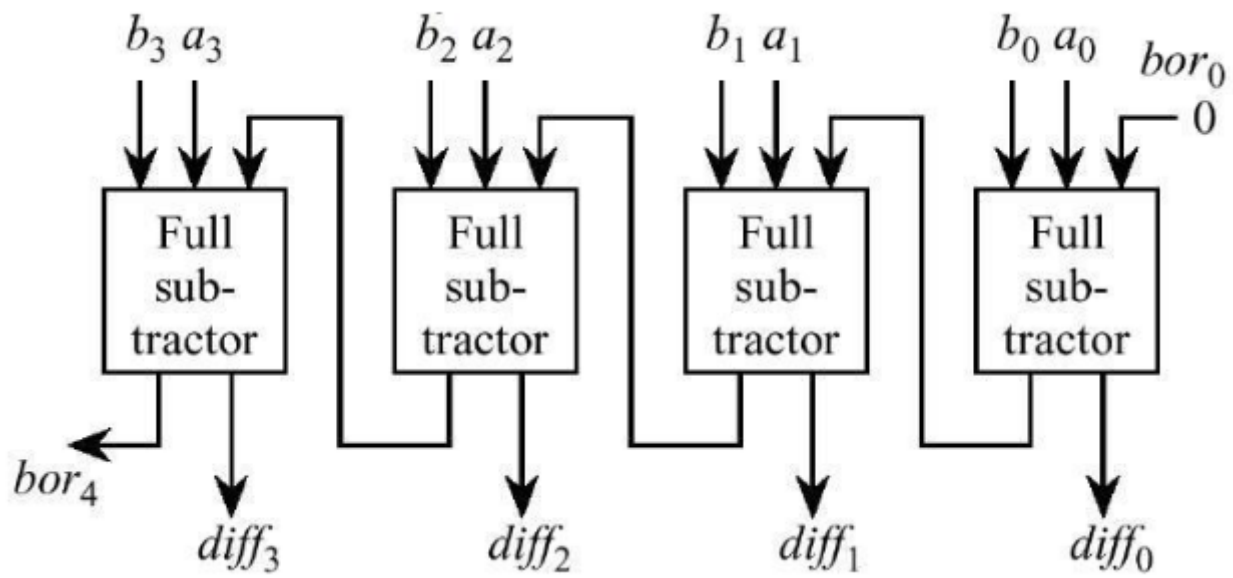


**Figure 2-2:** Full Subtractor

### Truth Table for Full Subtractor:

A	B	Bin	D	Bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Like the full adder, the full subtractor has an additional input, **Bin** (borrow in), which comes from the **Bout** of another full subtractor. By using full subtractors, a **Ripple Borrow Subtractor** can be created by connecting **Bout** from the less significant bit to **Bin** of the more significant bit.



**Figure 2-3:** Ripple Borrow Subtractor

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